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T.Y.B.Sc. (Sem.-VI) Examination March - 2026

Mathematics Paper : XX - MTH - 604 - Old Course Real Analysis - IV

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions	Seat No.:
(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book (2) Figures to the right indicate marks of corresponding question. (3) Follow usual notations. (4) Use of non-programmable scientific calculator is allowed.	Student's Signature

Q.1. Answer the following as directed : (Any FIVE) **10**

(1) Define Limit Point of Set and Closure of a Set.
(2) Show that $\{20\} \cup [\,2,6\,]$ is a closed subset of the metric space $\mathbb{R}^1$.
(3) Show that $\{20, 26\}$ is not a connected set in the metric space $\mathbb{R}^1$.
(4) Find the diameter of set $(20, 26)$ in the metric space $\mathbb{R}_d$.
(5) Justify: $[0,20\,] \cup \{26\}$ is not the complete subset of the metric space $\mathbb{R}^1$.
(6) Give an example of a complete metric space. Define Contraction Mapping.
(7) Is the metric space $\mathbb{R}_d$ compact? Justify your answer.
(8) Define Open Covering of Metric Space.

Q.2. Answer any Two of the following : **10**

(a) Prove ; for the subset E of a metric space $\langle M, \rho \rangle$; that : (i) $E \subseteq \overline{E}$.
(ii) $x \in \overline{E} \Rightarrow$ there exists $B[x\,;\,r]$ such that $B[x, r] \cap E \neq \phi$.
(b) Define Closed Set. Prove that every finite set in any metric space is closed.
(c) Let f be a continuous real - valued function on the metric space M.
Let A be the set of all $x \in M$ such that $f(x) \geq 0$. Then prove that A is closed.

Q.3. Answer any Two of the following : **10**

(a) If A_1 and A_2 are connected subsets of a metric space $\langle M, \rho \rangle$
and $A_1 \cap A_2 \neq \varnothing$, then prove that $A_1 \cup A_2$ is also connected.
(b) Define Bounded Set in Metric Space $\langle M, \rho \rangle$. If A is a totally
bounded subset of the metric space $\mathbb{R}_d$, then prove that A contains
only a finite number of points.
(c) Define Totally Bounded Set in Metric Space $\langle M, \rho \rangle$. Prove that every
finite set in any metric space is totally bounded.

Q.4. Answer any Two of the following : **10**

(a) If A is a closed subset of a complete metric space $\langle M, \rho \rangle$, then prove that $\langle A, \rho \rangle$
is complete.
(b) Define Complete Metric Space. Prove that the metric space $\mathbb{R}_d$ is complete.

(c) Define Fixed - Point. Prove that a mapping

$$T : \left\langle \left(0, \frac{1}{3}\right), |\cdot| \right\rangle \to \left\langle \left(0, \frac{1}{3}\right), |\cdot| \right\rangle$$

defined by $Tx = x^2$; for every $x \in \left(0, \frac{1}{3}\right)$; is contraction.
Show that T has no fixed - point.

Q.5. Answer any Two of the following : 10

(a) Define Compact Metric Space. If the metric space M is compact, then prove that every sequence of points in M has a subsequence converging to a point in M.

(b) Let A be a closed subset of a compact metric space $\langle M, \rho \rangle$. Then prove that $\langle A, \rho \rangle$ is compact.

(c) State Heine - Borel property. Prove that every finite set in any metric space is compact.